PARTICLE SWARM OPTIMIZATION IN COMPARISON TO GENETIC ALGORITHM FOR UNCERTAINLY- BASED OPTIMIZATION OF PIPE NETWORKS UNDER TRANSIENT FLOW CONDITIONS

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ABSTRACT

The integer discrete particle swarm is used as an optimization tool for the least cost design of water distribution systems under hydraulic uncertainty evaluated by the chance constraint formulation with different levels of uncertainty in demand. Transient flow conditions are considered. The transient states in the network are initiated by pump power failure and sudden valve closure. A transient analysis program combined with chance constraint model to evaluate the network uncertainty. The model is applied to the two-source network with a pump and a valve that has previously been analyzed under the same conditions but with genetic algorithm as the optimization tool and the results are compared to test the capability of the developed integer discrete particle swarm optimization model over genetic algorithm to search for the global minimum in the solution space. The results showed a better capability of the model over genetic algorithm to reach a least cost design (optimal pipe diameters) of water distribution systems under the predefined conditions and hence improving the search for effective and economical hydraulic protection strategies.

Keywords: Water Distribution Systems, Particle Swarm, Genetic Algorithm, Optimization, Chance Constraints, Uncertainty, Transient Flow.

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Nomenclature

a	Wave speed
A	Cross-sectional area of the pipe
$c_i(D_i)$	Cost of pipe *i* with diameter D_i per unit length
C_{P-SS}	Penalty cost in case of steady state
C_{P-WH}	Penalty cost in case of water hammer
$C_{P-WHMAX}$	Penalty cost in case of water hammer when $H_{j,max} > H_{max,TR}$
$C_{P-WHMIN}$	Penalty cost in case of water hammer when $H_{j,min} < H_{min,TR}$
C_T	Network total cost
D_i	Diameter of pipe *i*
D_{max}	Maximum diameter
D_{min}	Minimum diameter
E_p	Energy supplied by a pump
$f_{i,j}$	Friction factor of pipe
g	Gravitational acceleration

h_f	Head loss due to friction in a pipe
H	Piezometric head above arbitrary datum
H_j	Pressure head at node *j*
H_k	Pressure head at node *k* of a pipe *i* divided into segments
$H_{k,\max}$	Maximum pressure head at all nodes *k* in a divided pipe *i*
$H_{k,\min}$	Minimum pressure head at all nodes *k* in a divided pipe *i*
$H_{\max,TR}$	Maximum allowable pressure head for transient conditions
$H_{\min,ST}$	Minimum allowable pressure head for steady state
$H_{\min,TR}$	Minimum allowable pressure head for transient conditions
H_p	Head delivered by pump
$L_{i,j}$	Length of pipe
N_{nodes}	Total number of nodes in the network (excluding reservoirs and tanks)
N_{parts}	Number of segments into which the pipe is divided
N_{pipes}	Total number of pipes
$P[\,]$	Probability
$q_{i,j}$	Flow rate in the pipe connecting nodes *i* and *j*
Q_j	External demand at node *j*
Q_p	Pump discharge
t	Time
V_i	Steady state velocity in pipe *i*
W_{ss}	Penalty multiplier for the steady state condition
W_{wh}	Penalty multiplier for the water hammer condition
x	Distance along the pipe axis
Z	Objective function
α_j	Uncertainty level for the demand at node *j*
μ_{Q_j}	Mean nodal demand
σ_{Q_j}	Standard deviation of nodal demand
ϕ	Cumulative distribution function
$\phi[\,]$	Standard normal distribution function

Subscript

i,j	connection between nodes *i* and *j*

Abbreviation

COV_Q	Coefficient of Variation of nodal demand
GA	Genetic Algorithm
PSO	Particle Swarm Optimization

1 INRTODUCTION

Continuous research is still going on seeking improvement of the evolutionary optimization techniques especially particle swarm optimization (PSO) to be more and more efficient to search for the global optimal solution without any violation in the hydraulic constraints, which is the main concern of design engineers. **Ezzeldin et al. (2014)** developed the integer discrete particle swarm optimization model, (IDPSO*net*), with a new boundary condition (billiard boundary condition) that remarkably enhanced the performance of PSO as an optimization technique by improving its capability to search for the optimal solution. The model application to two benchmark networks under steady state conditions showed such capability by obtaining the minimum function evaluation number (F.E.N.) as a measure for the minimum computational efforts when the least cost is the same or the global optimal cost when applied to a large-scale network. **Jung et al. (2006)** stated that any optimized design which fails to properly account for water hammer effects is likely to be at best, suboptimal and, at worst, completely inadequate. Another motivation for exploring transient issues that is becoming progressively important is water quality. In reality, since all pipeline systems leak and since hydraulic transients occur almost continuously in distribution systems, low pressure transient waves draw untreated and possibly hazardous water into a pipeline system (**Karim et al., 2003**; **Jung and Karney, 2003**). It then becomes important to test the capability of the developed IDPSO*net* model to handle water distribution networks working under transient flow conditions. The use of uncertainty-based optimization techniques in the least-cost design of water distribution networks under transient conditions has received little attention. **Djebedjian (2009)** studied the uncertainty-based optimization of water distribution networks under water hammer events. The genetic algorithm was used as the optimization tool and the chance constraint formulation for incorporating the demand uncertainty in the network design. The present paper aims at applying the developed IDPSO_{net} model to test its capability as uncertainty-based optimization model to analyze networks under water hammer conditions. The results are then compared to that obtained by **Djebedjian (2009)** using genetic algorithm as the uncertainty-based optimization tool.

2 OPTIMIZATION MODEL FORMULATION

The initial network design cost is the objective to be minimized in the present study. For a given network layout and demand requirements, the minimization of cost aims to find the optimal pipe diameters of the network.

Therefore, the objective function used in this study is the total cost of the given network. The total cost C_T is calculated as:

$$C_T = \sum_{i=1}^{N_{pipes}} c_i\left(D_i\right)\cdot L_i \tag{1}$$

where N_{pipes} is the total number of pipes, $c_i\left(D_i\right)$ the cost of pipe *i* with diameter D_i per unit length and L_i the length of pipe *i*.

The optimization of water distribution systems is a constrained optimization problem. The constraints used in such systems for both steady and transient states are the design constraint and hydraulic constraints in steady and transient state are given respectively as:

$$D_{\min} \le D_i \le D_{\max} \qquad i = 1,\ldots,N_{pipes} \tag{2}$$

Steady State:

$$H_j \ge H_{\min,ST} \qquad j = 1,\ldots,N_{node} \tag{3}$$

Transient State:

$$H_{\min,TR} \le H_k \le H_{\max,TR} \quad k=1,\dots,N_{parts} \tag{4}$$

where N_{nodes} is the total number of nodes (excluding the sources as reservoirs and tanks), N_{parts} number of segments into which the pipe is divided, $D_{\min}$ minimum allowable diameter, $D_{\max}$ maximum allowable diameter, H_j pressure head at node *j*, $H_{\min,ST}$ minimum allowable pressure head for steady state, H_k pressure head at node *k* of a pipe *i* divided into segments, $H_{\max,TR}$ maximum allowable pressure head for transient conditions, and $H_{\min,TR}$ minimum allowable pressure head for transient conditions.

In optimization techniques, external penalty functions have been used to convert a constrained optimization problem into an unconstrained problem. Therefore, for the network optimization, the objective function *Z* is given as:

$$Z = C_T + C_{P-SS} + C_{P-WH} \tag{5}$$

where C_{P-SS} is the penalty cost in case of steady state, and C_{P-WH} penalty cost in case of water hammer.

The penalty cost in case of steady state is presented as,

$$C_{P-SS} = \begin{cases} 0 & \text{if } H_{\min,ST} - H_j \le 0 \\ \left[W_{ss} \cdot \sum_{j=1}^{N_{nodes}} \left(H_{\min,ST} - H_j \right) \right] & \text{otherwise} \end{cases} \tag{6}$$

where W_{ss} is the penalty multiplier for the steady state condition.
The total penalty cost in case of water hammer is described as follows:

$$C_{P-WH} = C_{P-WHMAX} + C_{P-WHMIN} \tag{7}$$

The penalty costs $C_{P-WHMAX}$ and $C_{P-WHMIN}$ are given as, **Ezzeldin (2014)**:

$$C_{P-WHMAX} = \begin{cases} 0 & \text{if } H_{k,\max} - H_{\max,Tr} \le 0 \\ \left[W_{wh} \cdot \sum_{i=1}^{N_{pipes}} \left(H_{k,\max} - H_{\max,Tr} \right) \right] & \text{otherwise} \end{cases} \tag{8}$$

$$C_{P-WHMIN} = \begin{cases} 0 & \text{if } H_{\min,Tr} - H_{k,\min} \le 0 \\ \left[W_{wh} \cdot \sum_{i=1}^{N_{pipes}} \left(H_{\min,Tr} - H_{k,\min} \right) \right] & \text{otherwise} \end{cases} \tag{9}$$

where $H_{k,\max}$ and $H_{k,\min}$ are the maximum and minimum pressure heads at all nodes *k* in a divided pipe *i*, and W_{wh} is the penalty multiplier for water hammer condition.

3 STEADY STATE ANALYSIS

Under steady state conditions and for a total number of nodes N_{node} in the network, the conservation of mass can be expressed as:

$$\sum_{j} q_{i,j} = Q_j \qquad j = 1,\ldots,N_{node} \tag{10}$$

where Q_j is the known demand at node *j*, and $q_{i,j}$ is the flow rate in the pipe connecting nodes *i*, *j*.

The conservation of energy, according to which the total head loss around any loop must equal to zero or is equal to the energy delivered by a pump, E_p, if there is any:

$$\sum h_f = E_p \tag{11}$$

In the general form, the head loss can be determined using either Darcy-Weisbach or Hazen-Williams resistance formulae in the form:

$$h_f = Kq^{\lambda} \tag{12}$$

where *K* is the loss coefficient of pipe and *q* is the pipe flow. The loss coefficient is expressed by the Hazen-Williams friction formula as:

$K = \omega \dfrac{L}{C^{1.852} D^{4.8704}}$ and □ = 1.852 (□ is a numerical conversion constant, which depends on the units used). *C* is the Hazen-Williams roughness coefficient of pipe.

4 CHANCE CONSTRAINT METHOD

The chance-constrained formulation was introduced by **Charnes and Cooper (1959)** and offers an efficient framework to model uncertainties in numerous applications in water resources management. It deals with uncertain RHS's (right hand side) assuming the decision maker is willing to make a probabilistic statement about the frequency with which constraints need to be satisfied.

The application of the chance constrained programming for the optimization problem with the objective function *Z* and inequality constraint of Eq. (10): $\sum_{j} q_{i,j} \geq Q_j$, can limit such constraint to a desired minimum probability level of compliance. Namely, the probability, $P\lfloor\ \rfloor$, of a constraint being satisfied is greater than or equal to a pre-specified value α_j:

$$P\left[\sum_{j} q_{i,j} \geq Q_j\right] \geq \alpha_j \tag{13}$$

or

$$P\left[Q_j \leq \sum_{j} q_{i,j}\right] \geq \alpha_j \tag{14}$$

where α is a parameter which determines the desired level of uncertainty.
Therefore, the optimization problem can be expressed as, **Lansey et al. (1989)**:
Objective function *Z* is to minimize the total design cost:

$$Z = \min C_T = \sum_{i=1}^{N_{pipes}} c_i(D_i) \cdot L_i$$

(15)

Subject to the constraints: Eqs. (2), (3), (4) and (13).

The future nodal demand is represented by normal random variables with mean, μ_{Q_j}, and standard deviation, σ_{Q_j}, as: $Q_j \sim N(\mu_{Q_j}, \sigma_{Q_j})$. By subtracting the average value μ_{Q_j} from both sides of the inequality of Eq. (14) and dividing both sides by the standard deviation (σ_{Q_j}) then the constraint becomes:

$$P\left[\frac{Q_j - \mu_{Q_j}}{\sigma_{Q_j}} \le \frac{\sum_j q'_{i,j} - \mu_{Q_j}}{\sigma_{Q_j}}\right] \ge \alpha_j$$

(16)

Eq. (16) can be rewritten in a simplified form as:

$$\phi\left[\frac{\sum_j q'_{i,j} - \mu_{Q_j}}{\sigma_{Q_j}}\right] \ge \alpha_j \tag{17}$$

where ϕ is the cumulative distribution function and $\phi[\ ]$ is the standard normal distribution function.

The final deterministic form of the constraint Eq. (14) is now written as:

$$\frac{\sum_j q'_{i,j} - \mu_{Q_j}}{\sigma_{Q_j}} \ge \phi^{-1}(\alpha_j) \tag{18}$$

Therefore, the nodal demand constraint can be written as a simple bound constraint as:

$$\sum_j q'_{i,j} \ge \mu_{Q_j} + \sigma_{Q_j}\,\phi^{-1}(\alpha_j) \tag{19}$$

For a given α_j, the nodal demand is calculated using the equality of Eq. (19).

The final deterministic chance constraint model for water distribution networks is given by the objective function Eq. (15) subject to the constraints Eqs. (2), (3) and (19). The model is nonlinear because of the nonlinear objective function, Eq. (15), and the non linear constraint, Eq. (19), for every node. The other constraints given by Eq. (2) for every pipe and Eq. (3) for every node are considered to be simple bound.

The coefficient of variation of nodal demand is defined as the standard deviation of demand divided by the mean of demand:

$$COV_Q = \frac{\sigma_Q}{\mu_Q} \tag{20}$$

From Eqs. (19) and (20), the relationship between the new nodal demand used in the calculations, $Q_{j,new} = \sum_j q'_{i,j}$, and the mean nodal demand, coefficient of variation of nodal demand, and inverse of cumulative distribution function is given as:

$$\frac{Q_{j,new}}{\mu_{Q_j}} \ge 1 + COV_Q \cdot \phi^{-1}(\alpha_j) \tag{21}$$

5 BASIC DIFFERENTIAL EQUATIONS OF UNSTEADY FLOW

The one-dimensional unsteady flow in closed conduits is described by the two quazi-linear hyperbolic partial differential equations, continuity and momentum, in the form, Wylie and Streeter (1993):

Continuity equation:

$$a^2 \frac{\partial V}{\partial x} + \frac{1}{\rho}\frac{\partial p}{\partial t} = 0 \tag{22}$$

Momentum equation:

$$\frac{\partial V}{\partial t} + \frac{1}{\rho}\frac{\partial p}{\partial x} + g\frac{dz}{dx} + \frac{f}{2D}V|V| = 0 \tag{23}$$

where the velocity, *V*, and the pressure, *p*, are the two dependent variables, space, *x*, and time, *t*, are the two independent variables. Other variables *a*, *f*, □, and *D* are system parameters independent of the time *t*.

5.1 The developed Particle Swarm Optimization model, IDPSO*net* model

Ezzeldin et al. (2014) developed a combined simulation-optimization model IDPSO*net*, (Integer Discrete Particle Swarm Optimization net), which has been written in FORTRAN language. IDPSO*net* model depends on two main techniques: The integer discrete particle swarm optimization technique to produce the optimal diameters, and the Newton-Raphson method to hydraulically simulate the network using the *H*-equations solution method (**Larock et al., 2000**).

The developed IDPSO*net* program permits the uncertainty-based optimization of water network subjected to several events that can cause rapid transients. These include the power failure at any number of pump stations, sudden valve closure at either end of any number of pipes, staged valve closure at the downstream end of any number of pipes and the sudden demand changes at any number of junctions. In this study, the transients resulting from pump power failure and sudden valve closure are considered.

6 CASE STUDY

The case study shown in Fig. 1 is based on the hypothetical network of **Larock et al. (2000)**; and previously analyzed by **Djebedjian (2009)** using genetic algorithm as the optimization tool. A pump power failure and a sudden valve closure are chosen to characterize the transient performance of the system.

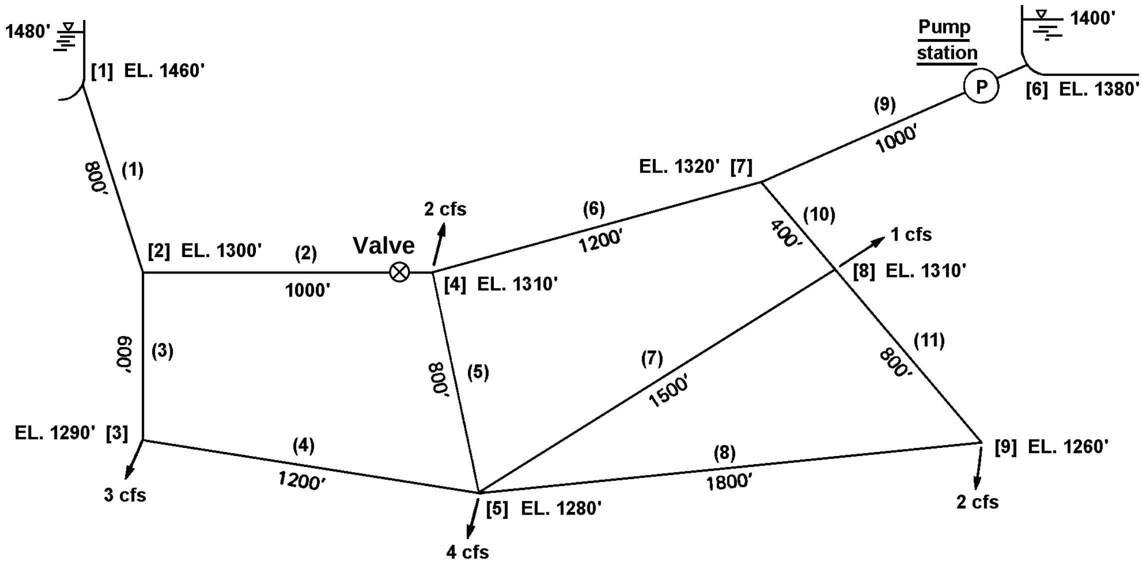


Figure1. Two-source network with a pump and a valve, Larock et al. (2000)

7 RESULTS AND DISCUSSION

7.1 Pump Power Failure

the results obtained for the water hammer event caused by the pump power failure are mentioned. Tables 1, 2 and 3 give the optimal cost and pipe diameters of best solutions for COV_Q = 10%, 20% and 30% of nodal demand for both IDPSO*net* and $GACCWH_{net}$.

The optimal solutions of COV_Q = 10%, Table 1, illustrates two exceptional equal results at □ = 0.5 and □ = 0.6. The optimal costs and diameters are identical. This may be attributed to the suitability of that optimal design to support the increased nodal demand when □ = 0.6. Also, it can be observed that 7 pipes have 6 in. diameter which is the minimum available diameter. The unavailability of smaller diameter may be the reason for that equality of optimal diameters as the IDPSO*net* is restricted by those minimum diameters.

From Tables 1, 2, and 3 for different COV_Q^s of 10%, 20% and 30% with uncertainty level□ ranging from 0.6 to 0.95, the application of IDPSO*net* in most cases (almost 56%) resulted in reaching less cost than obtained by genetic algorithm. Compared to the genetic algorithm, the IDPSO*net* showed remarkable capability in searching for the global solution when optimizing networks under water hammer conditions.

The comparison between IDPSO*net* and GACCWH*net* results due to cost ratio ($Cost_{PSO}/Cost_{GA}$) and F.E.N. ratio (FEN_{PSO}/FEN_{GA}) is shown in Fig. 2. It is evident that when the cost ratio is equal unity, the F.E.N. ratio is less than unity.

Table 1. Optimal costs and pipe diameters (in inches) for different required network reliabilities at COVQ = 10% for the pump power failure

Pipe ID	IDPSO*net* (Present study) □						GACCWH*net* (Djebedjian, 2009) □					
	0.50	0.60	0.70	0.80	0.90	0.95	0.50	0.60	0.70	0.80	0.90	0.95
1	15	15	15	15	15	15	15	15	15	15	15	15
2	12	12	12	12	12	12	12	12	12	12	12	12
3	8	8	8	8	10	10	8	8	10	8	10	10
4	6	6	6	6	6	8	6	6	6	6	6	6
5	6	6	8	8	8	6	6	6	6	10	8	6
6	6	6	6	6	6	6	6	6	6	6	6	6
7	6	6	6	6	6	6	6	6	6	6	6	6
8	6	6	6	6	6	6	6	6	6	6	6	6
9	6	6	6	6	6	6	6	6	6	6	6	6
10	10	10	8	10	10	10	10	10	10	8	10	10
11	6	6	6	6	6	6	6	6	6	6	6	6
Cost (*10^2)	2505	2505	2545	2585	2645	2685	2505	2505	2565	2625	2645	2685
FEN	1365	1085	2162	3181	1707	1283	1973	1224	3014	2671	1883	1529

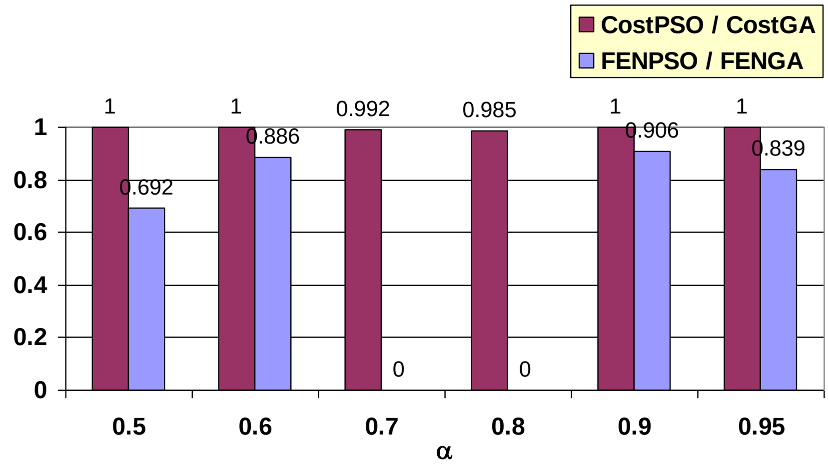
Table 2. Optimal costs and pipe diameters (in inches) for different required network reliabilities at COVQ = 20% for the pump power failure

Pipe ID	IDPSO*net* (Present study) α						GACCWH*net* (Djebedjian, 2009) α					
	0.50	0.60	0.70	0.80	0.90	0.95	0.50	0.60	0.70	0.80	0.90	0.95
1	15	15	15	15	15	15	15	15	15	15	15	15
2	12	12	12	12	12	12	12	12	12	12	12	12
3	8	8	8	10	12	10	8	8	10	8	12	10
4	6	6	6	8	8	8	6	6	8	6	8	8
5	6	8	10	6	6	6	6	8	6	8	6	6
6	6	6	6	6	6	8	6	6	6	8	6	8
7	6	6	6	6	6	6	6	6	6	6	6	6
8	6	6	6	6	6	6	6	6	6	6	6	6
9	6	6	6	6	6	6	6	6	6	6	6	6
10	10	8	8	10	10	10	10	10	8	10	10	10
11	6	6	6	6	6	6	6	6	6	6	6	6
Cost (*10^2)	2505	2545	2625	2685	2745	2805	2505	2585	2645	2705	2745	2805
FEN	1365	1252	1715	4688	1536	1566	1973	1855	1579	1934	1679	1726

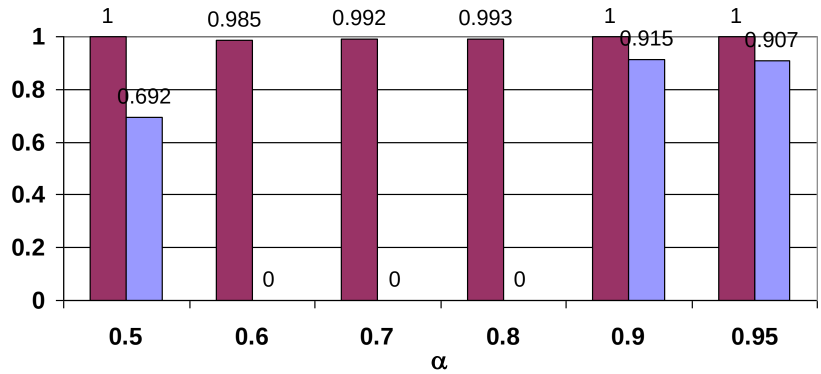
Table 3. Optimal costs and pipe diameters (in inches) for different required network reliabilities at COV_Q = 30% for the pump power failure

Pipe ID	IDPSO*net* (Present study)						GACCWH*net* (Djebedjian, 2009)					
	α						α					
	0.50	0.60	0.70	0.80	0.90	0.95	0.50	0.60	0.70	0.80	0.90	0.95
1	15	15	15	15	15	15	15	15	15	15	15	15
2	12	12	12	12	15	15	12	12	12	15	15	15
3	8	8	10	12	10	8	8	10	12	8	10	12
4	6	6	8	8	6	6	6	6	8	6	6	6
5	6	8	6	6	10	10	6	8	6	8	10	12
6	6	6	6	6	6	8	6	6	6	6	6	6
7	6	6	6	6	6	6	6	6	6	6	6	6
8	6	6	6	6	6	6	6	6	6	6	6	6
9	6	6	6	6	6	6	6	6	6	6	6	6
10	10	10	10	10	8	10	10	8	8	10	8	8
11	6	6	6	6	6	6	6	6	6	6	6	6
Cost ($*10^2$)	2505	2585	2685	2745	2885	2985	2505	2605	2705	2785	2885	3025
FEN	1365	3181	1658	2850	1571	4625	1973	1740	1236	1380	1992	2713

(a) COV = 10%



(b) COV = 20%



(c) COV = 30%

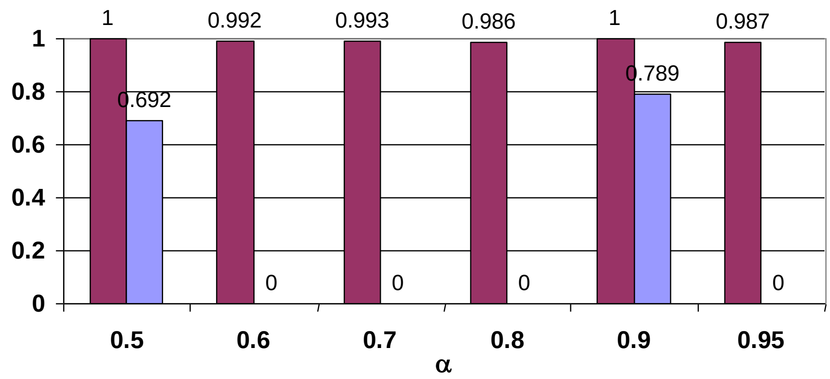


Figure 2. Cost and F.E.N. ratios versus α for COV_Q = 10%, 20% and 30% for the pump power failure

The results of the optimal costs for both IDPSO*net* and GACCWH$_{net}$ are shown in Fig. 3. The two trends of optimal cost versus uncertainty level □ for constant COV_Q and optimal cost vs. COV_Q for constant □ are the same as in Fig. 3. Generally, the optimal costs are greater than the steady state indicating that the design of water distribution networks to be reliable under water hammer event caused by pump power failure is more expensive. It is worth mentioning that the decision maker can prefer the employment of specialized surge control devices or the utilization of the optimal pipe design with pressure heads in the allowable range.

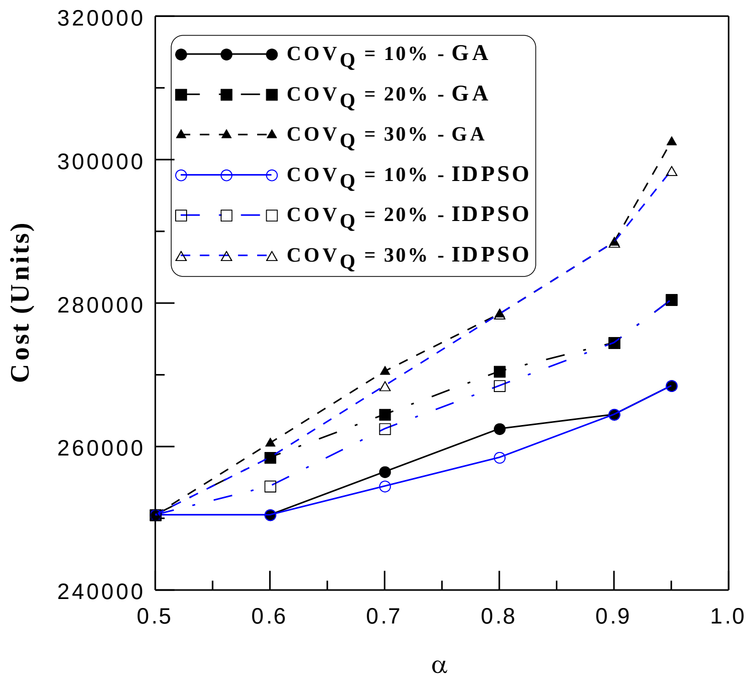


Figure 3. Total cost versus uncertainty α for COV_Q = 10%, 20% and 30% for the pump power failure

7.2 Sudden Valve Closure

The optimal costs and optimal pipe diameters resulted from the IDPSO*net* and GACCWH$_{net}$ for the sudden valve closure for COV_Q = 10%, 20% and 30% at different values of α are mentioned in Tables 4, 5, and 6, respectively. From Tables 4, 5 and 6 for different COV_Q^s of 10%, 20% and 30% with uncertainty level α ranging from 0.5 to 0.95, the application of IDPSO*net* in most cases (almost 63%) resulted in reaching less cost than obtained by genetic algorithm. Compared to the genetic algorithm, the IDPSO*net* showed remarkable capability in searching for the global solution when optimizing networks under water hammer conditions.

Cost ratio and F.E.N. ratio of both IDPSO*net* and GACCWH*net* results for different COV_Q^s and α are shown in Fig. 4.

Table 4. Optimal costs and pipe diameters (in inches) for different required network reliabilities at COVQ = 10% for the sudden valve closure

Pipe ID	IDPSO*net* (Present study) α						GACCWH*net* (Djebedjian, 2009) α					
	0.50	0.60	0.70	0.80	0.90	0.95	0.50	0.60	0.70	0.80	0.90	0.95
1	8	8	8	8	8	10	8	8	8	8	8	8
2	6	6	6	6	6	8	6	6	8	6	6	8
3	10	10	10	12	12	15	10	10	10	12	12	10
4	6	8	8	6	8	10	8	6	8	6	8	6
5	6	6	6	6	8	6	6	6	6	6	8	6
6	12	10	10	10	12	10	10	10	10	10	12	12
7	6	6	6	6	6	6	6	6	6	6	6	6
8	6	6	6	6	6	6	6	6	6	6	6	6
9	12	12	12	15	12	12	12	15	12	15	12	15
10	8	8	8	8	10	6	8	8	8	8	10	12
11	6	8	8	8	6	6	8	6	8	8	6	10
Cost ($*10^2$)	2565	2645	2645	2785	2865	3005	2645	2645	2745	2785	2865	3105
FEN	4035	1832	5978	6897	4279	5833	3862	5231	4547	7130	7327	2515

Table 5. Optimal costs and pipe diameters (in inches) for different required network reliabilities at COVQ = 20% for the sudden valve closure

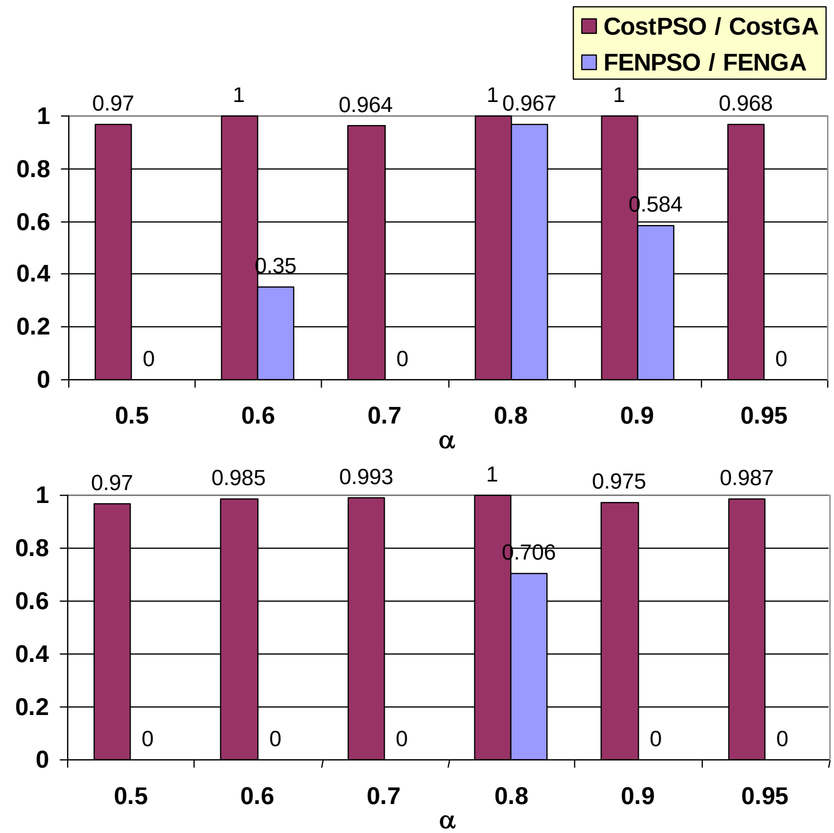
Pipe ID	IDPSO*net* (Present study) α						GACCWH*net* (Djebedjian, 2009) α					
	0.50	0.60	0.70	0.80	0.90	0.95	0.50	0.60	0.70	0.80	0.90	0.95
1	8	8	8	10	10	10	8	8	8	10	10	10
2	6	6	6	6	6	6	6	6	6	6	6	6
3	10	10	10	15	15	15	10	10	12	15	12	12
4	6	8	8	10	12	10	8	6	8	10	12	10
5	6	6	6	6	6	6	6	6	8	6	6	6
6	12	10	12	10	10	10	10	10	12	10	10	12
7	6	6	6	6	6	6	6	6	6	6	6	6
8	6	6	6	6	6	6	6	6	6	6	6	6
9	12	12	12	12	12	15	12	15	12	12	15	15
10	8	8	12	6	8	8	8	10	10	6	8	10
11	6	8	8	6	6	6	8	6	6	6	6	6
Cost ($*10^2$)	2565	2645	2845	2905	3065	3145	2645	2685	2865	2905	3145	3185
FEN	4035	8247	4891	5002	9356	12622	3862	6297	6082	7084	3094	5711

Table 6. Optimal costs and pipe diameters (in inches) for different required network reliabilities at COVQ = 30% for the sudden valve closure

Pipe ID	IDPSO*net* (Present study) α						GACCWH*net* (Djebedjian, 2009) α					
	0.50	0.60	0.70	0.80	0.90	0.95	0.50	0.60	0.70	0.80	0.90	0.95
1	8	8	10	10	10	12	8	8	10	10	10	12
2	6	8	6	6	8	6	6	6	6	6	6	6
3	10	12	15	12	15	15	10	10	15	12	12	15
4	6	8	12	10	10	12	8	8	12	10	12	15
5	6	6	6	6	6	6	6	6	6	6	6	6
6	12	10	10	15	10	12	10	12	10	12	15	10
7	6	6	6	6	6	6	6	6	6	6	6	6
8	6	6	6	6	6	6	6	6	6	6	6	6
9	12	12	10	12	15	15	12	12	10	15	12	15
10	8	10	6	8	8	6	8	12	6	12	10	6
11	6	8	6	6	6	8	8	8	6	6	6	6
Cost ($*10^2$)	2565	2845	2925	3185	3245	3505	2645	2845	2925	3225	3345	3545
FEN	4035	3749	6012	9649	9571	24178	3862	4632	6822	5396	6889	3897

(a) COV = 10%

(b) COV = 20%



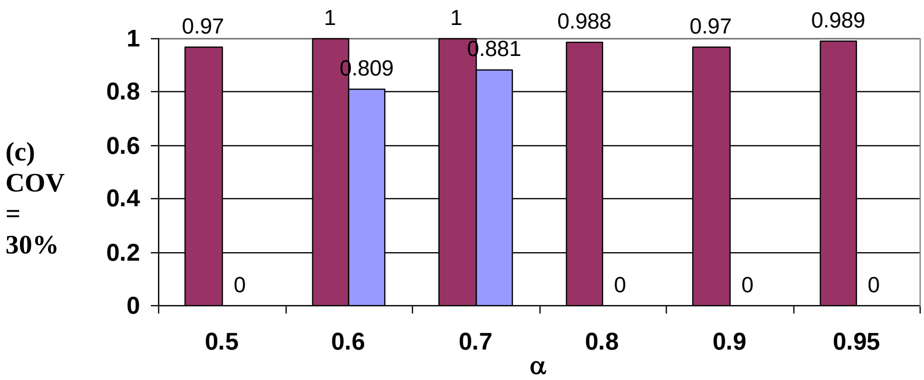


Figure 4. Cost and F.E.N. ratios versus α for COV_Q = 10%, 20% and 30% for sudden valve closure

Figure 5 illustrates the results of the optimal costs for both IDPSO*net* and $GACCWH_{net}$.
The general trends for the relationship between the cost and the uncertainty level are in agreement with uncertainty level α as in Figs. 3 and 5 but the curves are not smooth as in the previous ones indicating the difficulties faced by the PSO to find other better solutions for this more complicated case. The comparison of the uncertainty-based optimization for water hammer event caused by sudden valve closure with that of pump power failure indicates that the optimal costs in the case of sudden valve closure are higher than that in the case of pump power failure, e.g. at α = 0.5 the optimal cost of optimization is 256,500 units with an increase of 2.4% than that for pump power failure. Also, the variation in the optimal diameters in the case of sudden valve closure is vast compared to that of the case of pump power failure.

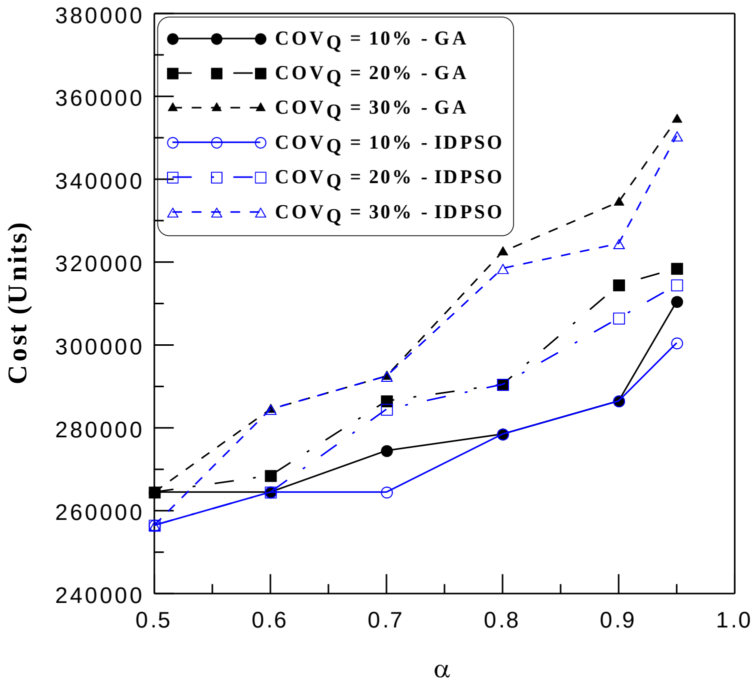


Figure 5. Total cost versus uncertainty α for COV_Q = 10%, 20% and 30% for the sudden valve closure

8 COMPARISON OF OPTIMAL COSTS

The comparison of the optimal costs resulted from the uncertainty-based optimization using the IDPSO*net* model for steady state, pump power failure and sudden valve closure is shown in Fig. 6. The optimal cost ratio is defined as the ratio between the optimal cost for the studied case and the optimal cost for the steady state at α = 0.5 (218,500 units). The figure indicates generally that the optimal cost increases gradually from the steady state to the pump power failure to the sudden valve closure. The optimal cost ratios for optimization ($\alpha = 0.5$) for steady state, pump power failure and sudden valve closure are 1, 1.146 and 1.174, respectively. For COV_Q = 30% and $\alpha = 0.95$, the optimal ratios for these three cases are 1.211, 1.366 and 1.604, respectively. Whereas, the average optimal ratios calculated for 18 values of different COV_Q and α for each of steady state, pump power failure and sudden valve closure are 1.065, 1.216 and 1.321, respectively.

The results shown in Fig. 6 indicate the effectiveness of the selection of pipe diameters for surge protection and the related increase in total cost to respond to the increase in nodal demands. Although that this study is limited to two causes of transients, but the IDPSO*net* model has the capability of studying the other causes like sudden increase in a nodal demand and gradual close of valve with different rates.

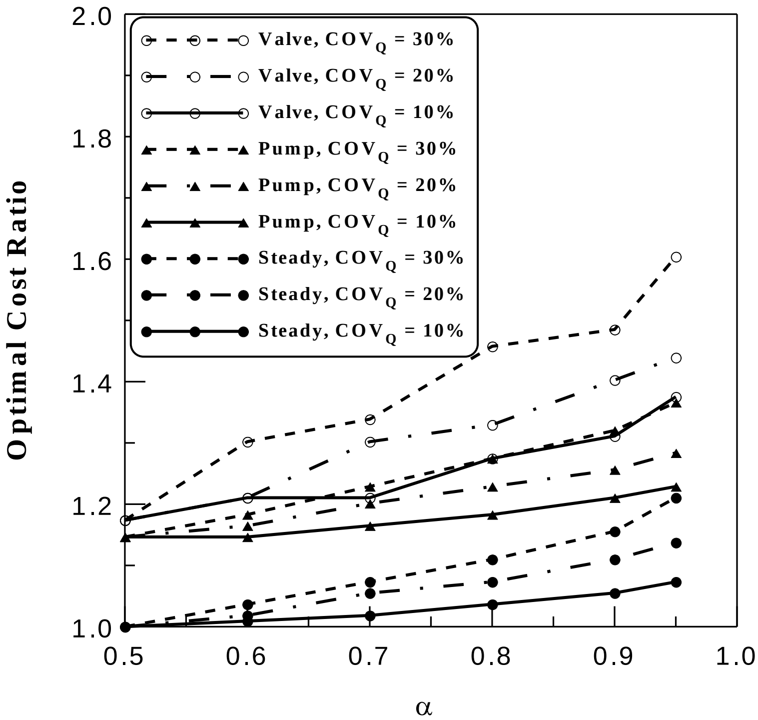


Figure 6. Optimal cost ratio versus α for COV_Q = 10%, 20% and 30% for the steady, pump power failure and sudden valve closure

CONCLUSIONS

The application of the IDPSO*net* model to the pre-specified network working under water hammer conditions revealed the following conclusions:

1. The application of the IDPSO*net* model with billiard BC under pump power failure resulted in reaching less cost than GA at many cases (56%), other cases are the same cost of GA but with F.E.N. ratio much less than unity.
2. The application of the IDPSO*net* model with billiard BC under sudden valve closure resulted in reaching less cost than GA at most cases (63%), other cases are the same cost of GA but with F.E.N. ratio much less than unity.
3. The IDPSO*net* model proved to be an efficient optimization tool when compared to GA in the optimization of water distribution systems considering both steady and transient conditions under uncertainty. The ratio of the F.E.N. which is defined as the value of PSO to its corresponding GA value is less than unity when reaching the same cost by

both PSO and GA which means better capability of the IDPSO*net* model than genetic algorithm.

4. The present model shows a remarkable capability of the robustness and efficiency with the billiard boundary condition to a better and rapid search for the optimal solution within the search space and proved to be a promising optimization tool within the frame of other optimization tools available in the literature.

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