A GIS SPATIAL INTERPOLATION METHOD FOR DETERMINING THE DRAWDOWN FIELD IN THE VICINITY OF MULTIPLE WELLS

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ABSTRACT

This research develops a useful GIS spatial interpolation method for determining the drawdown field in multiple well problems. The method is called Multiple Well Drawdown Interpolator (MWDI). The method implements Dupuit's formula to automatically determine drawdown at every grid pixel based on its relative location to the operating wells. The developed MWDI method outperforms the standard interpolation methods (IDW, SPLINE, and KRIGING) in groundwater interpolation from well data. The MWDI is fully automated and requires minimum effort from the user which is very favorable for engineering applications. It is expected to have different applications of MWDI including dewatering and optimum well positioning. The functionality of the MDWI method is demonstrated in this research on a case study from Egypt.

Keywords: Well; Ground Water ; Drawdown; GIS; Interpolation

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1 INTRODUCTION

The problem of drawdown interference due to wells is a common problem in groundwater hydrology. The estimation of the drawdown field in multiple wells problem is very essential for many applications such as dewatering and well design. This is important to ensure optimum well positioning and to properly select the types of submerged pumps and estimate the installation depths to avoid dry operation. In addition, it is important to estimate the composite drawdown at any location due to multiple wells operation in order to assess the appropriateness of a certain dewatering scenario. The problem can be studied using either the analytical superposition approach (Freeze and Cherry, 1979; Reilly et al., 1987; Bruggeman, 1999; Bakker and Hemker, 2002) or using the numerical modeling approach (Harbaugh and McDonald 1996; Harbaug et al., 2000; Hemker, 2004; Xu et al. 2012). Numerical models are increasingly employed to solve groundwater flow problems using either the finite difference or the finite element methods (MODFLOW, MicroFEM, MLAEM ...etc). Although the numerical modeling approach has more to offer in terms of the ability to model aquifer heterogeneity, it requires considerable effort to characterize the aquifer properties in three dimensions. For small scale problems, such heterogeneity is not significant and an analytical solution would be preferred if made much simpler. This paper proposes a GIS analytical interpolation method that can be used to estimate the drawdown field in multiple well problems. The main advantage of this method is its usage simplicity. The method can provide a rapid and acceptable solution for the drawdown field in small scale problems with much less design effort. In addition, it can provide a very useful visualization tool. It should be noted that the widely used GIS spatial interpolation methods (i.e., IDW, Spline, and Kriging) can not be used for drawdown interpolation from well data. This is because they can not consider the drawdown cone resulting from pumping the wells. The interpolation surfaces resulting from these three standard interpolation methods will only be influenced by the drawdown values in the wells giving much lower surfaces in between the pumping well. Details of the standard GIS spatial interpolation methods can be found in ESRI, 2005; Tsanis and Gad, 2001; and Wang and Wang, 2012. The inability of these interpolation methods to simulate the drawdown surface from well data gives rise to the importance of the method developed in this research.

2 THEORETICAL BACKGROUND

2.1 The Step Drawdown Method

Step-drawdown tests were first introduced by Jacob (1947) to study the influence that the discharge Q has on the steady state drawdown. Based on a number of drawdown tests, he concluded that the observed drawdown consists of two components in terms of Q (linear and nonlinear). He showed that the linear component represents both the linear aquifer head loss and the linear head loss in the well itself while the nonlinear term represents the additional turbulent head losses near the well screen. This interpretation led to describe the observed drawdown s_w in a pumped well by the general well loss equation:

$$s_w = AQ + CQ^2 \qquad \rightarrow \qquad (1)$$

Where A and C are the linear and nonlinear head loss coefficients. It is a common practice to fit Equation 1 to the step drawdown pumping data to determine the coefficients (Helweg, 1994; Kawecki, 2000; Rovey and Niemann, 2001; Barrash et al., 2006; and others)

2.2 Dupuit's Formulation

The solution for steady-state pumping of an ideal unconfined aquifer was developed by Dupuit (1857) one year after Darcy (1856) published the filtration law that later became Darcy's law. The Dupuit solution is for a homogeneous isotropic aquifer of uniform thickness over a horizontal impervious substratum when the well fully penetrates the aquifer. The Dupuit equation relates the constant flow rate, Q (steady-state conditions), to the saturated hydraulic conductivity, K, and either the saturated thickness h at radial distance r from the pumping well or the drawdown s defined as $s = H - h$, where H is the initial saturated thickness (i.e., the aquifer thickness):

$$h^2 = hw^2 + \frac{Q}{\pi K}\ln\left(\frac{r}{rw}\right) \qquad \rightarrow \qquad (2)$$

Where, hw is the saturated thickness (i.e., head) inside the pumped well and rw is well radius. Note that $s_w = H - h_{w,}$ where s_w is the steady state drawdown in the well. For a long time, the original equations of Dupuit have been viewed as approximations. Currently, it is known that the flow rate equation is exact when the unsaturated seepage (above the water table) is neglected (Chenaf and Chapuis, 2007). Many modifications have been made on Dupuit formula to account for partial penetration and for the seepage face near the well radius (Heinrich, 1964; Brauns, 1981; Gefell et al., 1994; Chapuis et al., 2001; Simpson et al., 2003; and others). Such modifications can be appended to the work presented in the current research.

3 THE MWDI

MWDI is a GIS-based analytical method (written using Visual Basic) that determines the composite drawdown in grid format based on the superposition of drawdown effects of the operating wells. The required input consists of the unconfined aquifer thickness *H*, hydraulic conductivity *K*, well radiuses *rw*, and the well loss coefficients *A & C*. In addition, the operating discharges are required to be supplied to MWDI. The MWDI incorporates two main modules. The first module is called the WELLCD (Well Composite DrawDown) and is responsible for determining the composite drawdowns inside the wells. The second module is called DGRIDGEN (Drawdown Grid Generator) which interpolates the well drawdowns into GIS grid format.

3.1 The WELLCD module

This module can be run in either controlled discharges or uncontrolled discharges modes.

The controlled discharges mode

In this mode, the discharges are known and supplied to the MWDI as input (Figure 1). To explain the method, consider a group of *n* wells. Let Q_a and Q_b denote known discharges at two different wells (where, $a = 1, 2, 3, \ldots\ldots, n$ and $b = 1, 2, 3, \ldots\ldots, n$). Let $s_{a,b}$ denotes the drawdown in well number *a* due to the pumping in well number *b* and l_{ab} denotes the distance between the two wells. The principal component drawdowns at the two wells (i.e., $s_{a,a}$ and $s_{b,b}$) can be obtained by plugging the known discharges into Equation 1 to get:

$$s_{a,a} = AQ_a + CQ_a^2 \quad \rightarrow \quad (3)$$

$$s_{b,b} = AQ_b + CQ_b^2 \quad \rightarrow \quad (4)$$

The drawdown in well *a* due to the pumping in well *b*, $s_{a,b}$, is obtained as follows:

$$h_{b,b} = H - s_{b,b} \quad \rightarrow \quad (5)$$

Using the coordinate system, the distance between wells is:

$$l_{ab} = \sqrt{(x_a - x_b)^2 + (y_a - y_b)^2} \quad \rightarrow \quad (6)$$

$$h_{a,b} = [hw_{b,b}^2 + \frac{Q_b}{\pi K} \ln(\frac{l_{ab}}{rw_b})]^{0.5} \quad \rightarrow \quad (7)$$

$$s_{a,b} = H - h_{a,b} \quad \rightarrow \quad (8)$$

Hence, the composite drawdown at well *a* , s_a , is given by:

$$s_a = \sum_{b=1}^{n} s_{a,b} \quad \rightarrow \quad (9)$$

Refer to Figure (1) for additional explanations of the above variables.

The uncontrolled discharges mode

This mode does not require the discharge values as input. The module intersects the pumps characteristic curves and the systems curves in order to find the operating discharges. Note that a system curve depends significantly on the drawdown value. The module applies the same equations shown above (equations 3 to 9) in a trial and error procedure to find the operating discharges. In order to explain this procedure, let the pump characteristic curve be represented in:

$$Q = f\{h_p\} \quad \rightarrow \qquad (10)$$

Where h_p and Q are the pump head and discharge respectively. The system curve can be given in:

$$h_{sys} = s + h_{static} + K'Q^2 \quad \rightarrow \qquad (11)$$

Where s is the composite drawdown in the well, h_{static} is the difference in elevation between the delivery level and the initial groundwater level, and $K'Q^2$ is the friction losses in the delivery system. The trial and error procedure runs as follows:

- Assume a set of starting values for the discharges ($Q_{assumed}$).
- Use the equations from 3 to 9 to calculate the composite drawdowns in the wells (s_a).
- Plug the drawdowns into equation 11 to calculate h_{sys}
- Use hp = h_{sys} in equation 10 to calculate a set of calculated discharges ($Q_{calculated}$)
- If $|Q_{assumed} - Q_{calculated}|$ < error threshold, then finish
- Else, take $Q_{assumed\ new} = (Q_{assumed} + Q_{calculated})/2$ and repeat

3.2 The DGRIDGEN MODULE

The DGRIDGEN module calculates the drawdown from multiple wells into a grid of cells. The user specifies the cellsize and grid extent among the input variables (refer to Figure 2). DGRIDGEN writes the drawdown values directly into the standard grid ASCII format that is readable by many software including GIS. Let Q_a denotes the operating discharges at each of the n wells (where, $a = 1, 2, 3, \ldots\ldots, n$). Consider a grid cell i at which the drawdown is to be calculated. $s_{a,a}$ is the principal drawdown in well a (i.e., as if well a is a single working well in the aquifer), $s_{i,a}$ is the drawdown at cell i due to pumping in well number a.

$$s_{a,a} = AQ_a + CQ_a^2 \quad \rightarrow \qquad (12)$$

$$hw_{a,a} = H - s_{a,a} \quad \rightarrow \qquad (13)$$

$$l_{ia} = \sqrt{(x_a - x_i)^2 + (y_a - y_i)^2} \quad \rightarrow \qquad (14)$$

$$h_{i,a} = [hw_{a,a}^2 + \frac{Q_a}{\pi K}\ln(\frac{l_{ia}}{rw_a})]^{0.5} \quad \rightarrow \qquad (15)$$

$$s_{i,a} = H - h_{i,a} \quad \rightarrow \qquad (16)$$

Hence, the composite drawdown at cell i , s_i , is given by:

$$s_i = \sum_{a=1}^{n} s_{i,a} \quad \rightarrow \qquad (17)$$

3.3 Technical Details

The MWDI is written using VB6 and compiled as a dynamic link library (mwdi.dll) that includes ESRICORE GIS library. This makes use of the broad built in GIS functions available in ESRICORE. In addition, the MWDI is computationally very efficient since it uses dynamic arrays (run time memory management) for storing grids in computer memory. After the model is installed, MWDI library is made available to ArcMap interface as a button. Once MWDI is invoked, the library is "hooked" to ArcMap and MWDI window opens and starts listening to the event analysis. Input to MWDI consists of one point shapefile that contains the pumping wells.

All well data is entered inside the attribute table of the shapfile (i.e., radiuses, discharges if known, and well loss coefficients). The remaining input is entered in MWDI window as shown in Figure (2).

MWDI consists of two main modules: WELLCD and DGRIDGEN that run in series (refer to section 3). Brief technical descriptions of the modules are presented here. The WELLCD module calculates the composite drawdown in the wells using either known discharges or using try and error to find the operating discharges. It uses dynamic arrays to hold the well coordinates, discharges, drawdowns, and head losses into memory. It then loops on the wells and add the drawdown components from other wells to find the composite drawdown. The calculated discharges and drawdowns are finally written to the attribute table of the wells shapfile. The DGRIDGEN constructs the output grid, opens a two dimensional dynamic array to hold grid parameters, loops on the grid cells to calculate the composite drawdown, and writes the values into an ArcInfo formatted ASCII grid file. It should be noted that if a cell is found to exist inside a well radius it is given the value of drawdown inside this well. The MWDI finally adds the output drawdown grid to ArcGIS data frame and writes the composite drawdowns and the corresponding discharges (if unknown) to the wells attribute table.

4 TEST CASE

The test case is a dewatering exercise where groundwater is required to be lowered by at least 7 meters along the sewer alignment shown in Figure (3). The initial groundwater is 6 m below ground surface (refer to equation 19). The unconfined aquifer thickness is 60 m with hydraulic conductivity of 0.14 m/hr. The step flow pumping test of the aquifer is shown in Figure (4) where best fitted well/aquifer loss parameters are A = 0.09 and C = 0.0016. Well radius is 0.14 m and the pump and system curves are given respectively by:

$$Q = -0.0016 h_p^{\,3} + 0.198 h_p^{\,2} - 8.85 h_p + 204 \qquad \rightarrow \qquad (18)$$

$$h_{sys} = s_a + 6 + 0.005 Q^2 \qquad \rightarrow \qquad (19)$$

Where Q is in m^3/hr and h is in meters. Note that Equation (18) is a third degree polynomial that fits the pump characteristics curve (pump model: VANSAN 15 Hp VSP0643 5 stages). Figure (5) presents the output of MWDI for two dewatering alternatives. It should be noted that the operating points for the pumps in alternative 1 (from north to south) are 61.84, 61, 60.7, 60.9, 61.6 m^3/hr. The operating points in alternative 2 are 63.5, 62.8, 62.9, 63.3 m^3/hr respectively from north to south. Figure (5) shows clearly the efficient uses expected for MWDI

CONCLUSIONS

A GIS analytical method for determining drawdown in multiple well problems has been developed. The method is called MWDI (Multiple Well Drawdown Interpolator) and has been implemented as a spatial interpolation method in the GIS environment. MWDI is a very efficient tool to study multiple well problems especially in dewatering applications. This is because the method is a quick method that requires minimum effort for the user as compared to numerical modeling. Using MWDI, the user can quickly study the different wells scenarios and find the optimum scenarios. In addition, the method can consider the effect of drawdown on the delivery system characteristics which affects the operating points of the submersible pumps. MWDI is strongly recommended as an analysis tool for small scale problems where heterogeneity in the unconfined aquifer is not significant.

REFERENCES

Bakker, M., Hemker, K. (2002) A Dupuit formulation for flow in layered, anisotropic aquifers. *Advances in Water Resources*, 25(7), 747-754.

Barrash, W., Clemo, T., Fox, J., Johnson, T. (2006) Field, laboratory, and modeling investigation of the skin effect at wells with slotted casing, Boise Hydrogeophysical Research Site. *Journal of Hydrology* 326, 181–198.

Brauns, J. (1981) Drawdown capacity of groundwater wells. In *Proceedings of the 10th International Conference on Soil Mechanics and Foundation Engineering*, 1, 391–396. Stockholm, Sweden: Balkema.

Bruggeman, G. A., 1999. *Analytical Solutions of Geohydrological Problems, Developments in Water Science*, 46, Elsevier.

Chapuis, R.P., D. Chenaf, B. Bussie`re, M. Aubertin, and R. Crespo (2001) A user's approach to assess numerical codes for saturated and unsaturated seepage conditions. *Canadian Geotechnical Journal* 38, no. 5: 1113–1126.

Chenaf, D., Chapuis, R.P. (2007) Seepage Face Height, Water Table Position, and Well Efficiency at Steady State. *Ground Water*, 45, (2), 168–177.

Darcy, H. (1856) Les Fontaines Publiques de la Ville de Dijon. Paris, France: Victor Dalmont.

Dupuit, J. (1857) Me´moire sur le mouvement des eaux dans les terrains perme´ables. Me´moire de´pose´ en 1857, rapporte´ aux Comptes Rendus des se´ances de l'Acade´mie des Sciences, Paris, tome LII, se´ance du 3 juin 1861.

ESRI (Environmental Systems Research Institute) (2005). Arc/Info- ArcDoc version 9.1. Users Guide.

Freeze, R.A. and J.A. Cherry (1979). *Groundwater*. Englewood Cliffs, NJ: Prentice Hall.

Gefell, M.J., G.M. Thomas, and S.J. Rossello (1994). Maximum water-table drawdown at a fully penetrating pumping well. *Ground Water* 32 (3), 411–419.

Harbaugh, A. W., Banta, E. R., Hill, M. C., McDonald, M. G. (2000) *MODFLOW-2000, the U.S. Geological Survey modular ground-water model—User guide to modularization concepts and the ground-water flow process*, U.S. Geol. Surv. Open File Rep., 00– 92, 121 pp.

Harbaugh, A.W., McDonald, M.G. (1996) *User's Documentation for MODFLOW-96, an Update to the U.S. Geological Survey Modular Finite-Difference Ground-Water Flow Model.* U.S. Geological Survey, Open-File Report 96-485.

Heinrich, G. (1964) EineNa¨herung fu¨r die Freie Spiegel-fla¨che beim Vollkommenen Brunnen. O¨sterreichische Wasserwirtschaft 16 (12), 15–20.

Helwag, OJ (1994) A general solution to the step-drawdown test. *Ground Water,* 32 (3).

Hemker, C. J. (2004). MicroFEM Development & Support. Internet site: http://www.microfem.com.

Jacob, C.E. (1947). Drawdown test to determine effective radius of artesian well. *Am. Soc. of Civ. Eng. Trans*. 112 (Paper 2321), 1047–1064.

Kawecki, M.W. (2000). Transient flow to a horizontal water well, *Ground Water*, 38 (6), 842–850.

Reilly, T.E., Franke, O.L., Bennett, G.D. (1987) *The Principle of Superposition and its Application in Ground-Water Hydraulics*. USGS, TWRI3B6.

Rovey, C.W., Niemann, W.L. (2001) Wellskins and slug tests: where's the bias? *Journal of Hydrology* 243, 120–132.

Simpson, M.J., Clement, T.P., Gallop, T.A. (2003). Laboratory and numerical investigation of flow and transport near a seepage-face boundary. *Ground Water,* 41 (5), 690–700.

Tsanis, I.K., Gad, M.A. (2001) A GIS precipitation method for analysis of storm kinematics. Environmental Modelling & Software 16(3), 273-281.

Xu, X., Huang, G., Zhanc, H., Qu, Z., Huang, Q. (2012) Integration of SWAP and MODFLOW-2000 for modeling groundwater dynamics in shallow water table areas. *Journal of Hydrology* 412–413(0), 170-181.

Wang, H., Wang, Y. (2012) Analysis of Spatial Interpolation Methods. In Recent Advances in Computer Science and Information Engineering. Ed. Qian, Z, Cao, L., Su, W., Wang, T., Yang, H. Springer Berlin Heidelberg. 129: 507-512.

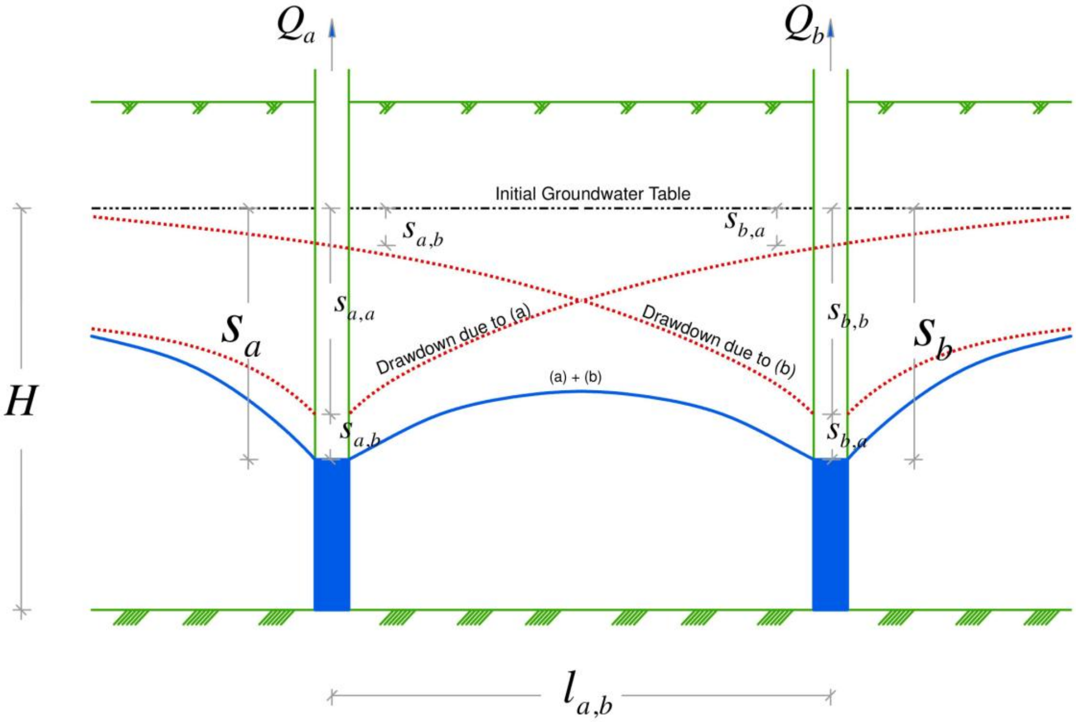
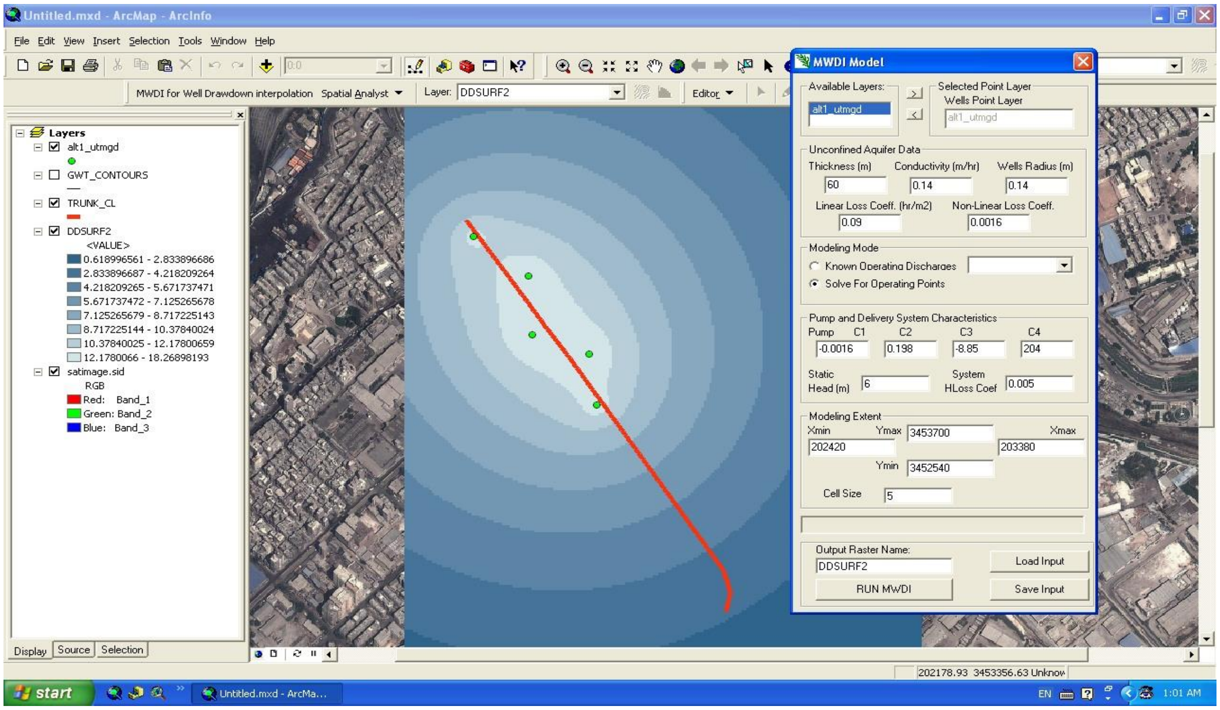


Figure 1Schematic representation of the variables used in MWDI.



Layer: DDSURF2

Layers
- ☑ alt1_utmgd
- ☐ GWT_CONTOURS
- ☑ TRUNK_CL
- ☑ DDSURF2
  - <VALUE>
  - 0.618996561 - 2.833896686
  - 2.833896687 - 4.218209264
  - 4.218209265 - 5.671737471
  - 5.671737472 - 7.125265678
  - 7.125265679 - 8.717225143
  - 8.717225144 - 10.37840024
  - 10.37840025 - 12.17800659
  - 12.1780066 - 18.26898193
- ☑ satimage.sid
  - RGB
  - 🟥 Red: Band_1
  - 🟩 Green: Band_2
  - 🟦 Blue: Band_3

Available Layers: alt1_utmgd
Selected Point Layer
Wells Point Layer: alt1_utmgd

Unconfined Aquifer Data
Thickness (m): 60
Conductivity (m/hr): 0.14
Wells Radius (m): 0.14
Linear Loss Coeff. (hr/m2): 0.09
Non-Linear Loss Coeff.: 0.0016

Modeling Mode
○ Known Operating Discharges
● Solve For Operating Points

Pump and Delivery System Characteristics
Pump C1: -0.0016
C2: 0.198
C3: -8.85
C4: 204
Static Head (m): 6
System HLoss Coef: 0.005

Modeling Extent
Xmin: 202420
Ymax: 3453700
Xmax: 203380
Ymin: 3452540
Cell Size: 5

Output Raster Name: DDSURF2

202178.93 3453356.63 Unknow

Figure 2The MWDI interface loaded into GIS.

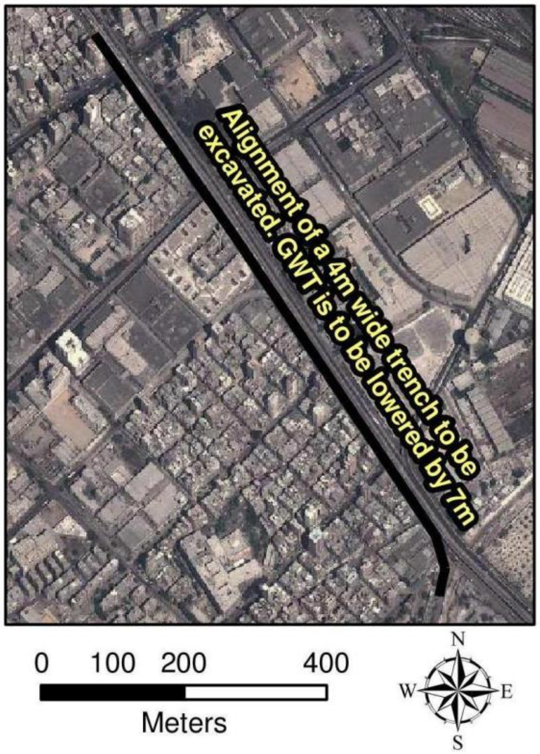


Figure 3 . A dewatering test case. The line represents the center line of a trench to be excavated for sewer installation. GWT is required to be lowered by a depth of 7 meters at least.

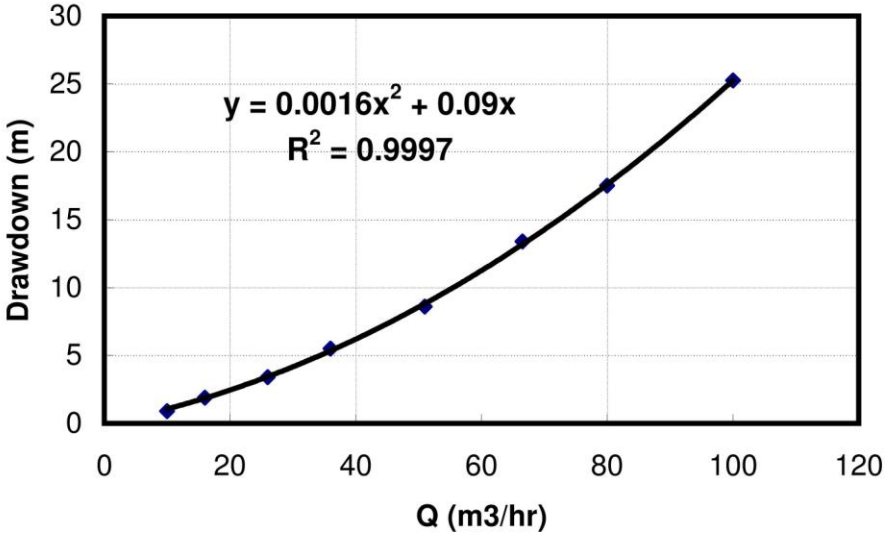


Figure 4. Fitting the well loss equation to the step flow pumping data.

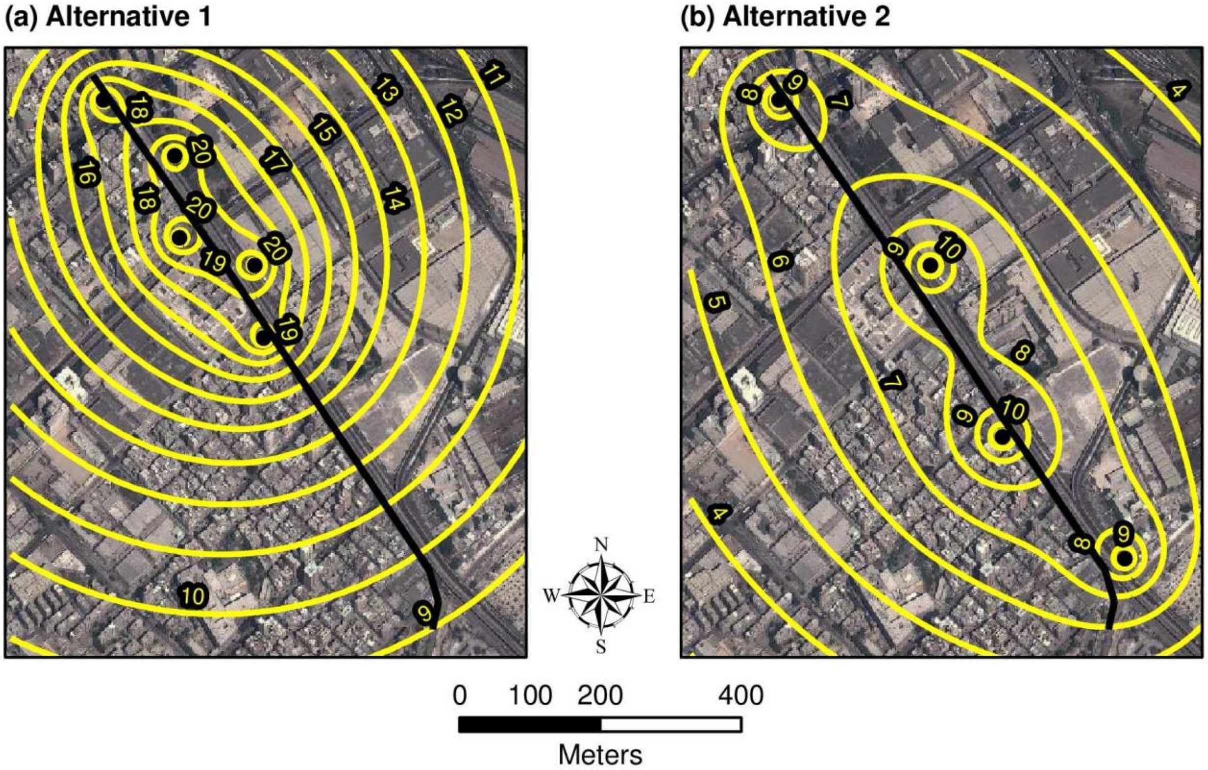


Figure 5. MWDI output for two dewatering alternatives.