NUMERICAL STUDY ON FLOOD ROUTING IN INDUS RIVER

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ABSTRACT

The flood routing is used for ascertaining the water level/discharge and flood hydrograph at different reaches of rivers/channels. The computation of flood propagation through numerical model is best tool of forecasting for flood management. This research paper presents the proposed development of a Finite Element model for flood routing using diffusive wave equation for computing flow in the river. The two-step semi-implicit Taylor-Galerkin technique based on Finite Element Method (FEM) was used for computing flood routing. The proposed FEM model is capable and stable of computing peak flow attenuation and time lag, which is vital to be computed for flood routing.

In this study, results of numerical model have been compared to analyze solution of observed data in Indus River for a reach between Guddu and Sukkur barrages. The accuracy of the proposed model was made by comparing the numerical predictions against the observed data. The statistical analyzed solution and the comparison result revealed that this model show a good agreement between computed and observed flow at upstream of Sukkur barrage of the river Indus.

Keywords: FEM, Flood routing, Flood Hydrograph, Indus River, Guddu-Sukkur reach

1. INTRODUCTION

The major flood events result from excessive flows within the Indus river basin, including the river Indus itself and main tributaries, namely the Jhelum, Chenab, Ravi, Sutlej and Kabul rivers. Melting snow from the mountains combined with monsoon rains (July to September) causes heavy floods. The worst flood disaster in the history of 80 years of Pakistan which occurred in July 2010 following heavy monsoon rains. The flood has resulted in loss of life and substantial damage to property, infrastructure and agriculture. UNESCO supported the national efforts to cope with the flood disaster in Pakistan [1]. In addition to above, UNICEF Pakistan responded quickly and effectively when the floods began to make their way down the length of the country[2].

High velocity current increases flood damage and economic use of land and the uncontrolled flood normally results destruction of properties [3]. If flood can be forecasted in advance then suitable warning and preparations can be taken to reduce the damages and loss of life [4]. Hence, flood routing models are necessary to be developed.

The Muskingum, a commonly used hydrologic routing, method is handling a variable discharge-storage relationship; models the storage volume of flooding in a river channel by a combination of wedge and prism storage [5]. This method was applied for flood routing for the Indus river reach between Guddu and Sukkur barrages for the peak flood duration from 28th July to 5th August, 1976; the results show minimum error between observed and estimated of 2.6%, however, the maximum is 16.2% which is really intolerable [6].

Analytical solutions of the basic equations are nearly impossible because of their non-linearly but numerical techniques may provide approximate solutions for some specific cases [7]. With significantly improved speed and capacity of digital computers in recent years' dynamic routing methods have been widely used for flood forecasting [8].

Although more hydrodynamic phenomenon in rivers is three-dimensional physical phenomena, but when the flow followed certain path, one-dimensional flow can be considered with accurate prediction. In this connection, mathematical models are useful tool in analysis of river behaviour and hydraulic structure [9].

For unsteady time-dependent flows, many finite element algorithms were developed such as Taylor-Galerkin, Characteristic Based Split (CBS), Least Square-Galerkin and such other schemes. The Taylor-Galerkin scheme generates accurate time-marching technique in accordance with high spatial resolution, which results from a Galerkin approximation [10].

The semi-implicit Tayler-Galerkin finite element algorithm was used for computing the fluid flow past a square obstacle in the center of a channel [11]. The performance of this numerical scheme is measured through speed-up and efficiency of distribution system. Predicted results encouraged to solve the large problems accurately and efficiently.

2. GOVERNING EQUATIONS

The Saint-Venant equation after neglecting inertia and momentum source terms in Saint–Venant system leading in the following equation for stream flow and overland flow.

$$\frac{\partial Q}{\partial t} = -C\frac{\partial Q}{\partial x} + D\frac{\partial^2 Q}{\partial x^2} \tag{1}$$

Where, Q = Discharge,

C = Wave celerity, D = Diffusivity or Diffusion Coefficient, t = time and x = Flow direction.

Non-Dimensionalization:

It is often convenient to cast the governing systems of equation in non-dimensional form using non-dimensional variables:

Space $x^* = \frac{Cx}{D}$, time $t^* = \frac{C^2 t}{D}$, inflow characteristics, $Q^* = \frac{Q}{Q_0}$ (where Q_o = Initial discharge).

$$\frac{\partial Q^*}{\partial t^*} = -\frac{\partial Q^*}{\partial x^*} + \frac{\partial^2 Q^*}{\partial x^{*2}} \tag{2}$$

Where, additionally, Q* = non-dimensionalized value of Q, t* = non-dimenisonalized value of t, and x* = non-dimensionalized value of x, Dropping the asterisks for simplicity and brevity, the above equation can be written as:

$$\frac{\partial Q}{\partial t} = -\frac{\partial Q}{\partial x} + \frac{\partial^2 Q}{\partial x^2} \tag{3}$$

Wave Celerity is velocity which develops variation moves along with the channel. The kinematic wave celerity can be expressed in terms of depth Y as under:

$$\text{Celerity (C)} = \frac{1}{B}\frac{dQ}{dY} \tag{4}$$

Where**, B** = width of the channel and Y = depth of water

Diffusivity or Diffusion Coefficient includes the combined effects of molecular diffusion turbulent mixing and the fluctuation of a flood occurrence is important parameter one-dimensional flow in rivers and streams. Indus River is given by [12] is used.

$$\text{Diffusion Coefficient} = D = \frac{0.058\,Q}{B\,S} \tag{5}$$

Where, S = Longitudinal bed slope.

3. NUMERICAL SCHEME

The choice of numerical scheme for numerical simulation depends on the accuracy, efficiency and stability of the algorithm. Literature shows that in an explicit scheme, the convergence rate is slow requires small time steps (Δt) which leads to use the alternate approach i.e. implicit and semi-implicit scheme. The implicit schemes enhance stability but computationally more expensive. Hence, Semi-implicit scheme can be used with the large time-steps are the numerical stable and efficient [13]. The finite element approach to discretising the problem divides domain/ region Ω of interest into sub-regions/ elements determined by some number of nodes. The finite element scheme is described in following two steps:

Step 1:

Predict the discharge Q at half time step (n+1/2) level using following equation

$$\left(\frac{2M}{\Delta t} + \frac{S_Q}{2}\right)\left(Q_j^{n+\frac{1}{2}} - Q_j^{n}\right) = -\left(N(C) + S_Q\right) Q_j^{n} + b.t. \tag{6}$$

Step 2:

Using the above information, correct the second order accurate Q at full time (n+1) level using following equation.

$$\left(\frac{M}{\Delta t} + \frac{S_Q}{2}\right)\left(Q_j^{n+1} - Q_j^{n}\right) = -N(C) Q_j^{n+\frac{1}{2}} - S_Q\, Q_j^{n} + b.t. \tag{7}$$

Where,

$$N(C) = \int_{\Omega} \frac{\partial \varphi_i}{\partial x} \varphi_i \varphi_j d\Omega = Non - Linear\ Convective\ Matrix$$

$$S_Q = \int_{\Omega} \left[\frac{\partial \varphi_i}{\partial x} \frac{\partial \varphi_j}{\partial x} \right] d\Omega = Diffusive\ Matrix$$

Here, b. t. = boundary terms, φ_j and ψ_j are shape functions, for more details ref. [14]

The numerical scheme was initially tested for three cases of the analytical solution for validating the accuracy of the above model through comparing predicted results with the available numerical results of Moussa and Bocquillon [15], which show a good agreement ref. [16].

4. INDUS RIVER: GUDDU TO SUKKUR REACH

Indus River is one of the main rivers of southern Asia and its source in Tibet, China. Indus flow traverses about 3100 km through India and Pakistan to the Arabian Sea. The river system's catchment area occupies 944,164 km^2; most of it is in Pakistan. The upper Indus, high in the Himalayas, is a surging torrent fed by melting snow water and monsoon rains. As it enters the Punjab plain in Pakistan, the Indus becomes sluggish and meandering. The Indus is joined by its main tributary, the Panjnad formed by the junction of the Chenab and Sutlej rivers and travels toward Arabian Sea. The Indus river system provides water for one of the largest irrigated areas in the world [17].

Indus river system comprises of seven rivers including the River Indus itself. The five rivers of Punjab i.e. Bias, Sutluj, Ravi, Chanab, and Jehlem discharge in Indus at Mithan Kot and the Kabul river from Afghanistan at Attock [18]. This river is perennial having annual volume of water more than 125 billion cubic meter (BCM). The average peak flow of this river is 14,000 cumecs. There are six head works at Indus River but out of them three Barrages Guddu, Sukkur and Kotri are in Sindh province territory.

The total length of Indus in Sindh is 864 km having Guddu-Sukkur reach 174 km (see Figure 1). The Guddu barrage is located at altitude 28^o 31’ 64” N and longitude 69^o 21’ 01” E and Sukkur barrage is located at altitude 27^o 44’ 47” N and longitude 68^o 56’ 58” E. The daily discharge data for the year 2002 at downstream of Guddu barrage and upstream of Sukkur barrage has been collected from Irrigation Department, Government of Sindh. The problem statement is shown in Fig. 2.

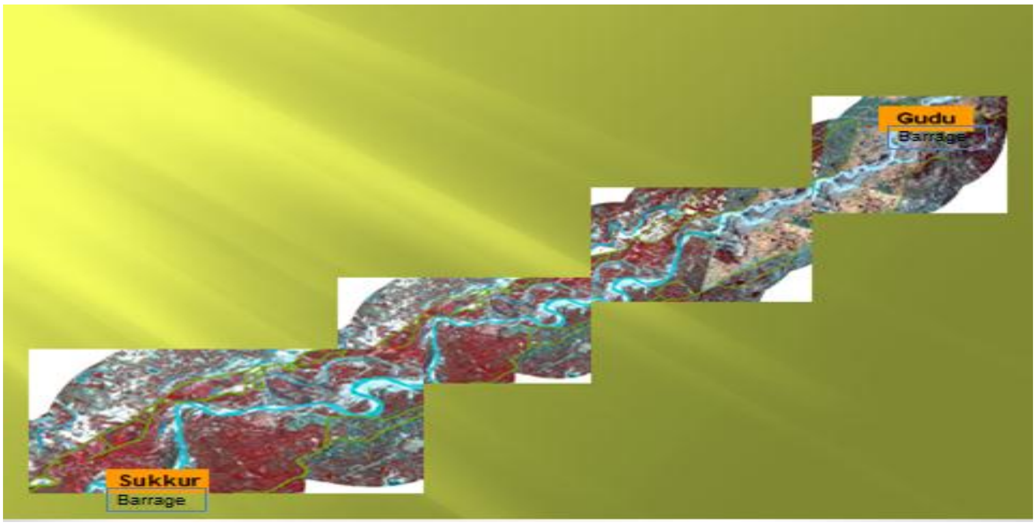


Fig. 1: Satellite imagery of study area (Indus River: Guddu-Sukkur reach)

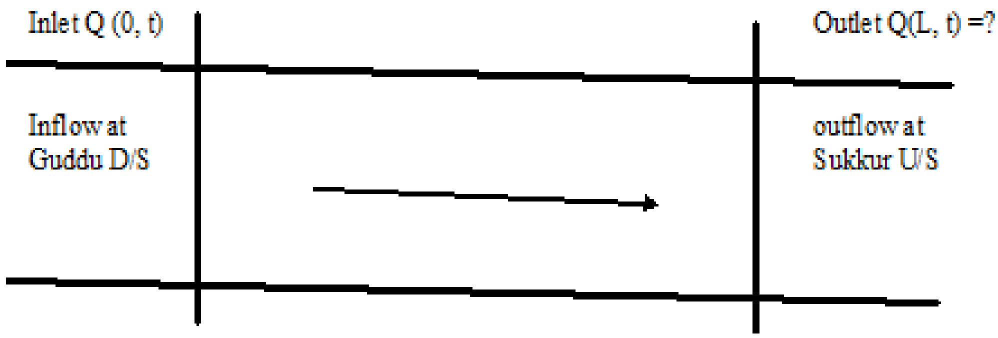


Fig. 2: Problem statement: Guddu-Sukkur reaches

The data of inflow hydrograph shown in Fig. 3 at downstream of Guddu barrage has been given to the proposed model as the inlet boundary condition. The numerical model described in equations (6 & 7) is applied to predict the outflow hydrograph at the upstream of Sukkur barrage. In this Guddu-Sukkur reach, no lateral inflow or outflow has been assumed.

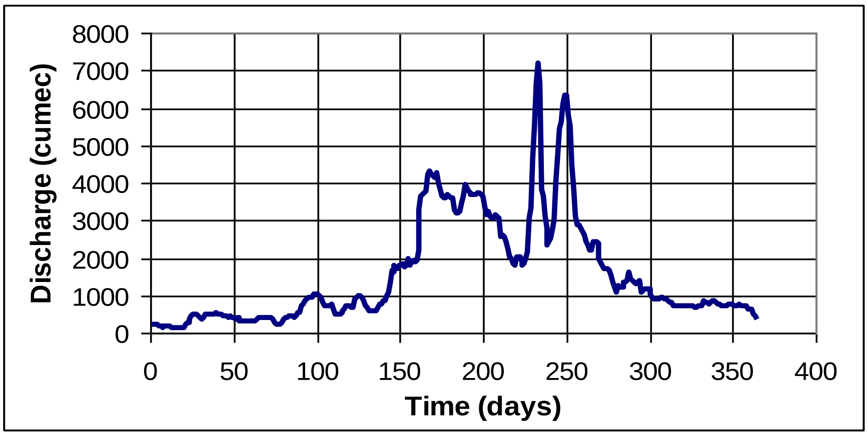


Fig. 3: Inflow hydrograph at downstream of Guddu barrage

5. RESULTS AND DISCUSSIONS

The model has been applied for the Indus River using daily discharges of 365 days of the year as initial condition at downstream of Guddu barrage and predicting flood routing, the behavior of water flows at upstream of Sukkur Barrage. Figure 4 shows inflow hydrograph at Guddu barrage (observed inflow) and comparison of observed and computed hydrograph at upstream of Sukkur barrage. From Fig. 4, it is clear that in the month of January water at the Guddu barrage is low that is less than 1000 cumecs in the reach up to mid of May (i.e. 140 days) and then gradually rise. There are total seven peaks, three small, two medium and two high; these four peaks i.e. medium and high have been observed in four months (i.e. from 150 to 270 days) which is due to monsoon rainfall and the snow melting (shown in Fig. 5).

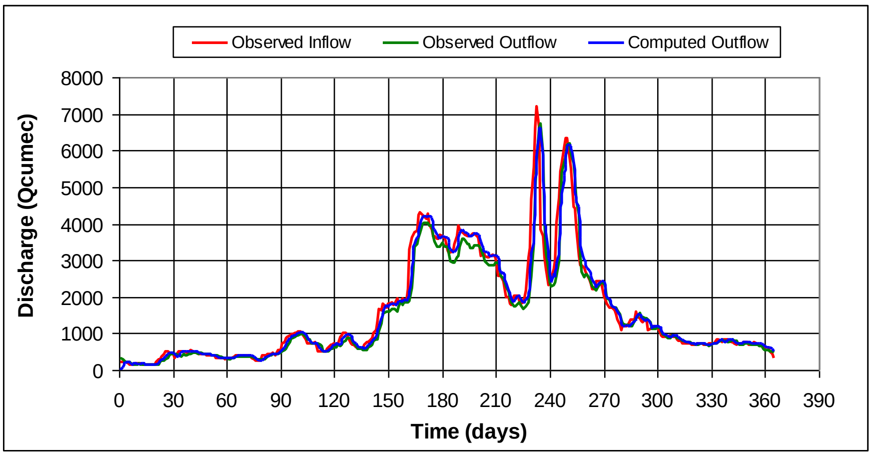
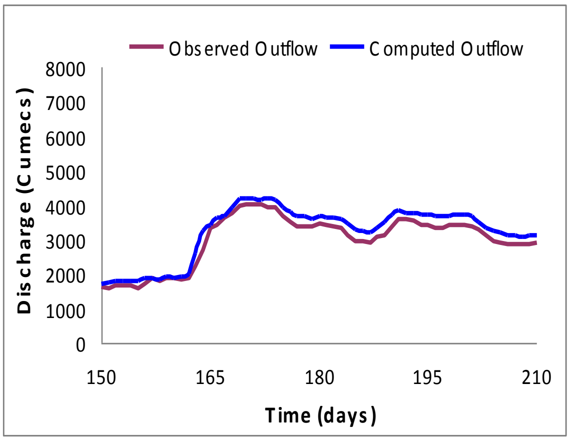
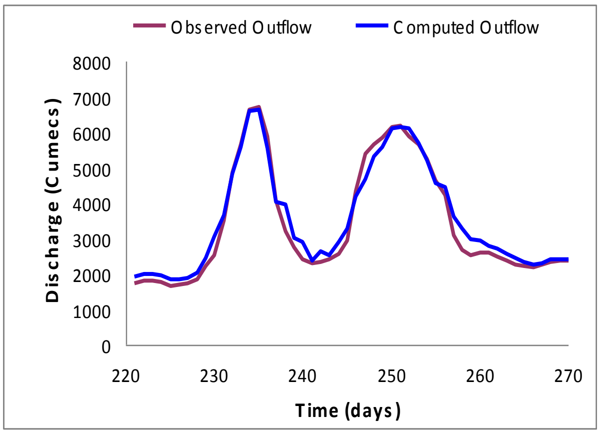


Fig. 4: Comparison of computed and observed outflow hydrograph at upstream of Sukkur barrage

Comparison of the observed and computed outflow hydrographs (i.e. at u/s of Sukkur barrage) shows a good agreement (Fig. 4). The absolute relative error for different months is varying from 2.1% to 10.4 %. However, the average annual relative error is 6.59 %. From above figure, it has been observed that there are seven peak flows, out of which four are the major peaks. To see the better performance of the model resolution with the observed data, four major peaks are shown separately in Fig. 5 (a and b)



(a)



(b)

Fig. 5: Comparison of computed and observed outflow hydrograph for four major peaks at upstream of Sukkur barrage

The observed and computed seven peak flows at u/s of Sukkur barrage is also shown in bar chart (see Fig. 6), which clearly show matching with each other in most of the cases. Their peak flow attenuations with respect to observed inflows are varying from 0.2 % to 7.3 %, while the average is 2.69 %.

The observed and computed outflows in terms of gain and loss with respect to inflow have been calculated for risings and recessions alternatively for all the seven peaks (Fig. 7). It has been observed that the maximum difference between computed and observed volumes of rising/recession is less than 0.5 BCM.

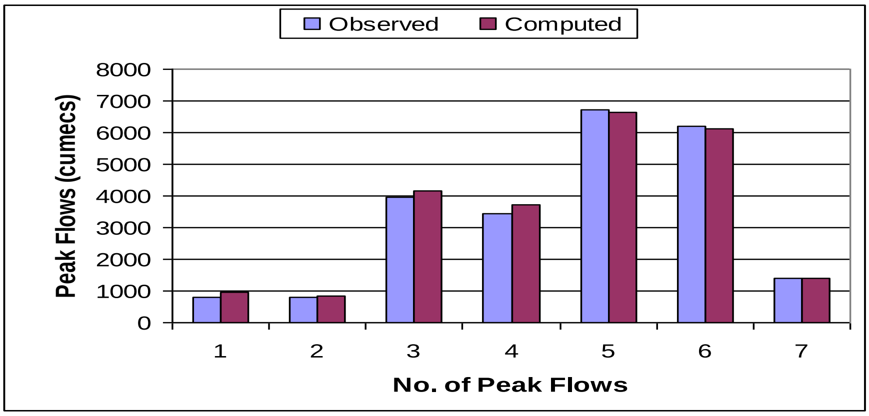


Fig. 6: Observed and Computed Peak Flows for Year 2002

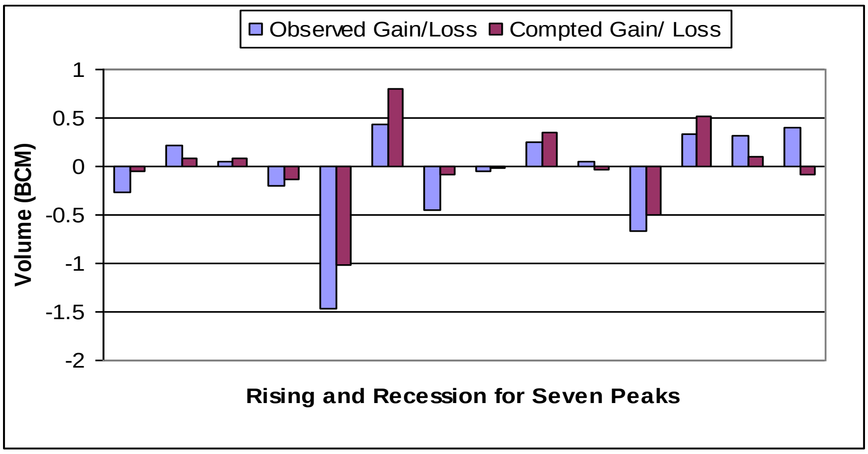


Fig. 7: Observed and Computed Gain/Loss in for Rising and Recession at upstream of Sukkur barrage

The observed and computed lag times are calculated and are shown in Fig. 8; three out of them are exactly each other. The observed minimum lag time is one day; however, the computed minimum lag time is zero (see Peak # 3). On the other hand, the maximum lag time for observed and computed is five days. Meanwhile, the average lag time is calculated which is two to three days.

The correlation coefficient R^2 of measured and predicted values of total volumes was calculated which comes 0.95 (see Table 1). This statistical value is an evidence for a nice agreement between observed and model results. The volume error is 0.467%, where as the peak error is about -1.33% respectively.

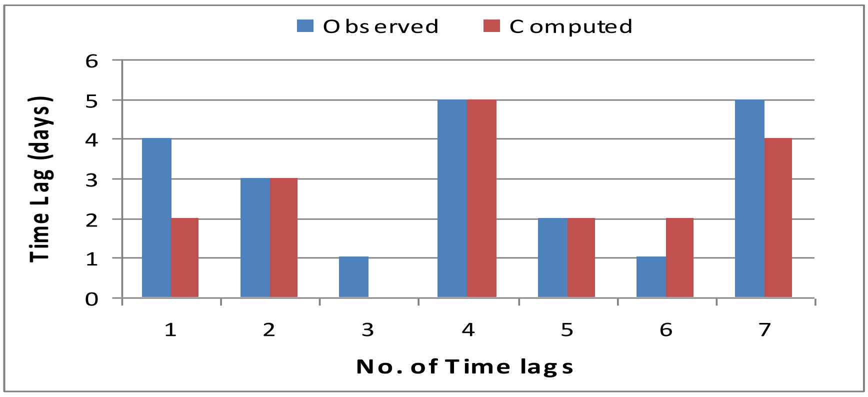


Fig. 8: Comparison of Computed and Observed Time Lag (days) for Year 2002

Table 1. Statistical Analysis of Measured and Modeled Data

Results	Value	Unit
Correlation coefficient R^2	0.95	-
Volume of observed flow	47.00766	BCM
Volume of simulated flow	49.314	BCM
Volume error	0.467	%
Observed Flow for Major Peak # 1	3947	m^3/sec
Computed Flow for Major Peak # 1	4156	m^3/sec
Error for Peak # 1	5.29	%
Observed Flow for Major Peak # 2	3441	m^3/sec
Computed Flow for Major Peak # 2	3704	m^3/sec
Error for Peak # 2	7.64	%
Observed Flow for Major Peak # 3	6728	m^3/sec
Computed Flow for Major Peak # 3	6640	m^3/sec
Error for Peak # 3	-1.31	%
Observed Flow for Major Peak # 4	6185	m^3/sec
Computed Flow for Major Peak # 4	6102	m^3/sec
Error for Peak # 4	-1.34	%
Maximum positive difference b/w observed & computed flow	1.342	m^3/sec
Minimum negative difference b/w observed & computed flow	-7.643	m^3/sec

The statistical analysis of measured and computed data reveals that there is good resolution between them. It also describes that result of the developed model is efficient and accurate which can be applied to any reach of the river.

6. CONCLUSIONS

The developed finite element model using semi-implicit predictor-corrector Taylor-Galerkin scheme is applied for the prediction of flood routing of Guddu-Sukkur reach of the Indus River. The observed and computed hydrographs at upstream of Sukkur barrage provides an accurate resolution of the model under field conditions. This model is capable of computing peak flow attenuation and time lag which is vital to be computed for flood routing.

Applicability and accuracy of this model has also been studied by statistical analysis. The correlation coefficient R^2 of measured and predicted data comes 0.95. The model shows accurate prediction with volume error of 0.47%, peak error of -1.325 % and maximum positive and negative difference of peak flow attenuation are 1.34 m^3/sec and -7.64 m^3/sec respectively. This model shows stability, accuracy and efficiency that is capable of solving diffusive wave equation for the selected Indus river reach.

ACKNOWLEDGEMENTS

The authors are thankful to the Institute of Water Resources Engineering and Management, Mehran University Engineering and Technology, Jamshoro, Pakistan, for providing facilities and Irrigation Department, Government of Sindh, Pakistan, for providing data to carry out this research work.

REFERENCES

[1] UNESCO. UNESCO. 2010. Section External Relation and Public Information United Nations Education, Scientific and Cultural Organization, printed by UNESCRO. Paris, France, March 2011.

[2] UNICEF. Pakistan Annual Report 2010. Unite for Children. United Nations International Children Emergency Fund, Islamabad, Pakistan, 2010

[3] Ilabuya, I.R., E. Atikpo, D.O Onaiwu, L.Umukoro and M.O Ezugwu. Application of Flood flow routing as a predictive Model for Flood Management and control, *Journal of Applied Technology in Environmental Sanitation*, vol.1, N.3, pp. 207 – 220, October 2011.

[4] Ming-Hsi Hsu, Jin-chengFu,Wen-cheng Liu. Flood routing with real-time stage correction method for flash flood forecasting in the Tanshiu River, Taiwan, *Journal of Hydrology*, vol. 283, pp. 267-280, 2003.

[5] Chow, V. T., Maidment, D. R., Ways, L. W. Applied Hydrology. *McGraw-Hill Book Co*. Singapore, 1988.

[6] 6Bhutto H. B., Qureshi M. A. and Lashari B. Flood Routing of the Indus River Gudu-Sukkur Reach", *Proceedings of International Seminar on Golden Jubilee of Sukkur Barrage* held in March, 1982 and published in *Special Issue of Bulletin of IDFCR Council*, vol. 12, No.1 & 2, June/December, 1982.

[7] Mehdi Azhdary Moghaddam, Behzad Firoozi. Development of Dynamic Flood wave Routing in Natural Rivers through Implicit Numerical Method, *American Journal of Scientific Research*, Issue 14, pp 6-17, 2011.

[8] Nguyen, Q.K., and Kawano, H. Simultaneous solution for flood routing in channel networks, *J. Hydraulic Eng*., vol. 121, No.10, pp. 744 -750, 1995.

[9] Delphi, M, Shooshtari, M.M and Zadeh, H.H. Application of Diffusion Wave Method for Flood Routing in Karun River, *International Journal of Environmental Science and Development*, Vol. 1, No. 5, pp 432-434, December, 2010.

[10] Donea. J., Giuliani S., Laval H. and Quartapelle L. Time-accurate solution of advection-diffusion problems by finite element, *Journal of Computation Methods Appl. Mech. Eng*., vol. 45, pp. 123-145, 1984.

[11] Baloch A. and Qureshi, A. L. Parallel Simulation of Fluid Flow past a Square Obstacle, *Mehran University Research Journal of Engineering and Technology*, Vol. 25, No. 1. January 2006.

[12] Keefer, T. N., and Mc Quivery, R. S. Multiple linearization flow routing method, *J. Hydraulics Division, ASCE*, vol. 100, No. 7, pp. 1031-1046, 1974.

[13] Solangi M. A., Baloch A. and Memon R. A. Influence of Blood Inertia on Vortex Enhancement in the Wake of Plaque Deposited Arteries. *Mehran University Research Journal of Engineering & Technology*, Vol. 30, No. 2, pp 213-224, Pakistan, April, 2011.

[14] Cuvelier, C., Segal, A. and Van Steenhoven, A. A. Finite Element Methods and Navier-Stokes Equations. *D. Reidol Publishing Company*, Dordrecht, Holland, 1986.

[15] Moussa, R., and Bocquillon, C., "Fractional-Step Method Solution of Diffusive Wave Equation", *Journal of Hydrologic Engineering, ASCE*, Vol. 6, No. 1, pp 11-19, 2001.

[16] Mahessar A. A., Baloch A. and Qureshi A. L. Solution of Diffusive Wave Equation using FEM for Flood Forecasting. *Submitted to Mehran University Research Journal of Engineering & Technology*, (under process), Jamshoro, Pakistan, 2012.

[17] The Indus River http://en.wikipedia.org/wiki/Indus_River

[18] Memon, A. A. Evaluation of Impacts on the Lower Indus River Basin Due to Upstream Water Storage and Diversion. *Proceedings, World Water & Environmental Resources Congress 2004*, American Society of Civil Engineers, Environmental and Water Resources Institute, Salt Lake City, Utah, June 27 - July 1, 2004.